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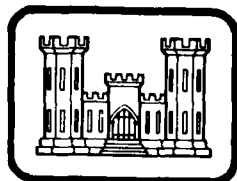FILMED
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RAYLEIGH WAVE VELOCITY MEASUREMENTS USING BROAD BAND FREQUENCY SOURCES

by

Otis Williams

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U. S. Army Engineer Waterways Experiment Station
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Final Report

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20. ABSTRACT (Continue on reverse side if necessary and identify by block number)

Rayleigh wave velocities were measured for broad band noise excitation of ground surfaces. Two noise sources were used: a vibrator using band-passed filtered white noise and a hammer impact for impulses. The Rayleigh wave velocities were computed using Fourier analysis to determine phase differences between transducers located a fixed distance apart. Measurements with the impulsive source agreed with the vibrator measurements and showed good results for

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20.   ABSTRACT (Continued).

Rayleigh wave velocities over wide frequency bands. The impulsive source is recommended as a more field-expedient method for computing Rayleigh wave velocity as long as the transducers used do not saturate. Best results can be obtained using the averages of repeated impulses.

PREFACE

This study was sponsored by the U. S. Army Corps of Engineers in furtherance of DA Project No. 4A762730AT42, "Terrain/Operations Simulations," Task B/E3, "Analytical Techniques for the Design and Application of Sensors."

This study was conducted by the U. S. Army Engineer Waterways Experiment Station (WES), Vicksburg, Miss., under the general supervision of Dr. John Harrison, Chief, Environmental Laboratory (EL), Mr. B. O. Benn, Chief, Environmental Systems Division (ESD), and Mr. J. R. Lundien, Chief, Battlefield Environment Team (ESD). The study was under the direct supervision of Dr. D. H. Cress. Project engineer was MAJ O. Williams. Personnel making significant contributions were SP5 G. Radu, ESD, and Mr. M. B. Savage, Instrumentation Services Division. SP5 Radu assisted in analyzing the data and developed the necessary computer programs. Mr. Savage assisted in collection of the field data. This report was prepared by MAJ Williams.

Commanders and Directors of WES during this study and preparation of the report were COL John L. Cannon, CE, and COL Nelson P. Conover, CE. Technical Director was Mr. F. R. Brown.

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CONTENTS

CONVERSION FACTORS, U. S. CUSTOMARY TO METRIC (SI)
UNITS OF MEASUREMENT

U. S. customary units of measurement used in this report can be converted to metric (SI) units as follows:

Multiply	By	To Obtain
inches	25.4	millimetres
pounds (force)	4.448	newtons
pounds (mass) per cubic foot	16.01846	kilograms per cubic metre

RAYLEIGH WAVE VELOCITY MEASUREMENTS USING
BROAD BAND FREQUENCY SOURCES

PART I: INTRODUCTION

Background

1. Rayleigh wave velocity data are often used to characterize the
near surface medium rigidity for such purposes as building and highway
foundation design and documenting test conditions for the evaluation of
military sensors that make use of seismic energy. Because of the close
relationship between Rayleigh wave and shear wave velocities, the
Rayleigh wave velocity can be used to estimate the shear modulus, and
is particularly accurate for obtaining such data near the surface.
Within several metres of the surface, the accuracy of the Rayleigh wave
velocity for estimating shear modulus is somewhat dependent on the com-
plexity of the layering of the medium (Lundien and Nikodem 1973).

2. The Rayleigh wave is a surface wave phenomenon whose degree
of influence decreases in an exponential fashion with depth beneath the
surface; its velocity is particularly sensitive to medium properties
within a wavelength of the surface. In order to characterize the shear
modulus very near to the surface (i.e. within approximately 20 cm),
which is often required in studies of the performance of battlefield
seismic sensors, it is necessary to obtain measurements of the Rayleigh
wave velocity for frequencies as high as 1000 Hz, i.e., frequencies con-
siderably higher than normally associated with seismic exploration tech-
niques for foundation design. In situ measurement of the shear modulus
of frozen ground is an example of a situation for which such high
frequencies are necessary in order to characterize the medium within
approximately the top 20 cm.

3. The traditional method for measuring Rayleigh wave velocities
has been to use a vibrator source-geophone layout where the vibrator
source has a driving frequency range from about 20 to 2000 Hz (Heukelom

and Foster 1960, Chang and Ballard 1973, Ballard and McLean 1975). The procedure for measuring the Rayleigh wave velocity has consisted of:

 a. Estimating the Rayleigh wave velocity for the medium under investigation for the lowest frequency available from the source (usually on the order of 20 Hz) and spacing the geophones so that the distance between geophones is on the order of a quarter of a wavelength.

 b. Sinusoidally exciting the medium at the lowest possible frequency for the source and measuring the time delay. The frequency is then increased incrementally (at the discretion of the operator) and the time delays are measured until the distance between geophones is on the order of a wavelength.

 c. Moving the geophones closer together (approximately a quarter of their previous spacing) and repeating the time delay measurement process. The velocities at each frequency are then computed from the geophone spacings and the measured time delays.

4. The above measurement technique is subject to several limitations:

 a. Field data collection and analysis for determining velocity as a function of frequency (with a frequency resolution of several Hertz) are extremely time-consuming for a broad frequency range.

 b. If the operator is not careful to properly monitor the wavelength versus geophone spacing relationships identified in 3a and b, large errors can be made during analysis because of failure to account for the presence of more than one wavelength between geophones.

 c. At high frequencies (exceeding several hundred Hertz), the wavelengths are so short that procedures 3a and b result in very close spacing of the geophones. In some cases, the geophone dimensions will not allow the geophones to be moved sufficiently close together to ensure that less than one wavelength occurs between geophones. Errors in locating the geophones can result in significant errors in velocity calculation.

5. The limitations identified above in measuring Rayleigh wave velocities using sinusoidal waves are of concern in site characterization for military seismic sensor studies as they normally must be carried out expeditiously in a variety of terrain conditions. A more appropriate method of measuring Rayleigh wave velocity has been

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developed that makes use of a multifrequency (broad band) source and conventional signal processing techniques; this report presents the results of applying such a method to obtain Rayleigh wave velocity data.

Purpose

6. The purpose of the study reported herein was to demonstrate the measurement of Rayleigh wave velocities using multifrequency (broad band) sources.

Approach

7. The source of vibrations coupled into the ground was programmed to provide a broad range of frequencies. Two sites having distinctly different constitutive properties were selected for conduct of tests. Geophone signatures induced by the broad band and impulse sources were recorded on magnetic tape, digitized, computer processed, and velocities as a function of frequency computed. These velocities were then compared with those measured using conventional techniques.

8. When the surface of a medium is disturbed by a source, wave motion can be initiated and coupled energy propagated outward. The mode of propagation can be Rayleigh (i.e. surface) waves, shear (transverse) waves, or compressional waves. As shown by Richart, Hall, and Woods (1970), the vibrational component induced by a vertical force on a medium surface is predominantly Rayleigh wave motion, particularly at distances on the order of several wavelengths or more from the source. For the Rayleigh wave motion, the amount of displacement from equilibrium is denoted by $\xi(r,t)$, where r is the range from the source to the measurement point and t is the time. Since the propagation is assumed to be radially outward, the amount of displacement arising from a single frequency source can be written for the Rayleigh wave as:

$$\xi(r,t) = A(r) \, \exp j \left\{ \omega \left[t - \frac{r}{v(\omega)} \right] + \theta \right\} \tag{1}$$

where

$A(r)$ = amplitude of the wave

$j = \sqrt{-1}$

ω = angular frequency

$v(\omega)$ = Rayleigh wave velocity

θ = arbitrary phase angle

The variables in parentheses, r and ω, denote a functional dependence on the respective variables. The quantity sensed by the geophones is the particle velocity $\dot{\xi}$, which is obtained by taking the time derivative of ξ. The resulting expression for $\dot{\xi}$ is given by:

$$\dot{\xi}(r,t) = \omega A \, \exp j \left\{ \omega \left[t - \frac{t}{v(\omega)} \right] + \theta + \frac{\pi}{2} \right\} \tag{2}$$

If two geophones are located at distances r_1 and r_2 from the source, the respective particle velocities at r_1 and r_2 are given by:

$$\dot{\xi}(r_1,t) = \omega A \, \exp j \left\{ \omega \left[t - \frac{r_1}{v(\omega)} \right] + \theta + \frac{\pi}{2} \right\} \tag{3}$$

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$$\dot{\xi}(r_2,t) = \omega A \exp j \left\{ \omega \left[t - \frac{(r_1 + \Delta r)}{v(\omega)} \right] + \theta + \frac{\pi}{2} \right\} \tag{4}$$

where

$$\Delta r = r_2 - r_1 \tag{5}$$

The propagation time Δt for the Rayleigh wave to move from r_1 to r_2 is given by:

$$\Delta t = \frac{\Delta r}{v(\omega)} \tag{6}$$

As may be seen in Equations 3 and 4, the above propagation time results in a difference in phase between the wave forms at r_1 and r_2. This phase difference $\Delta\theta$ is given by:

$$\Delta\theta = \frac{\omega \Delta r}{v(\omega)}$$

$$= \omega \, \Delta t \, (\omega) \tag{7}$$

where

$$\Delta\theta = \text{phase} \left\{ \dot{\xi}(r_1,t) \right\} - \text{phase} \left\{ \dot{\xi}(r_2,t) \right\} \tag{8}$$

The Rayleigh wave velocity is therefore given by:

$$v(\omega) = \frac{\omega \Delta r}{\Delta\theta} \tag{9}$$

The quantity $\Delta\theta$ is, in principle, obtainable from the measured data. However, mathematical computation of $\Delta\theta$ restricts its apparent values to a domain from $-\pi$ to $+\pi$ (the principal values of $\Delta\theta$ can be defined to be $0 \leq \Delta\theta \leq 2\pi$; however, most computer algorithms use the principal values in the range $-\pi \leq \Delta\theta \leq \pi$). If the propagation time over the distance Δr exceeds the period of the vibration (but is less than two periods), a value of 2π must be added to the apparent value

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of θ derived from mathematical computation.

9. The ambiguity concerning the true value of $\Delta\theta$ discussed above is a disguised version of the same problem encountered in the conventional procedure for Rayleigh wave velocity measurement in which the operator spaces the geophones no more than one wavelength apart (or corrects for such placement in subsequent data analysis). As shown in the next paragraph, the apparent ambiguity can be removed by using a broad band source (rather than a single frequency source) for Rayleigh wave velocity measurements.

10. For a broad band source having frequencies ranging from the tens of Hertz to the hundreds of Hertz, the phase difference at each frequency can be obtained by conventional cross-spectrum Fourier transform methods (see Appendix A). A phase difference plot as a function of frequency for a broad band source might appear as shown in Figure 1. For frequencies above approximately 75 Hz, an integral multiple of 2π must be added to the apparent phase difference if relative phase accuracy is to be preserved over the entire frequency range shown. Therefore, for the frequency range of 75 to 175 Hz, the total phase difference is the computed phase difference plus 2π. Similarly, for the frequency region of 175 to 275 Hz, the total phase difference is the computed phase difference plus 4π. The velocity at each frequency can be computed using the total phase difference and applying Equation 9.
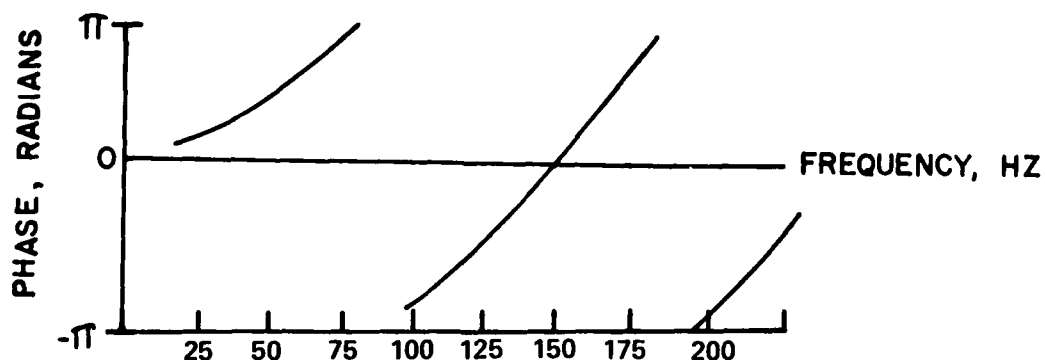

Figure 1. Phase versus frequency

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11. For the data collection effort, two kinds of sources were used: (a) a conventional electromagnetic vibrator driven b. a broad band noise source (as opposed to single frequency sine waves) and (b) an impulse induced by a hammer blow. The data collection equipment and procedures (including a description of the electromagnetic vibrator) are discussed in the following paragraphs. Two test sites were selected for the data collection effort, each having distinctly different media. One test site had a deep loess soil; the other had relative rigid surface material consisting of a soil-cement mixture overlying the deep loess soil. The test sites are discussed in the following paragraphs also.

Equipment

12. The equipment configuration for the Rayleigh wave velocity measurements is shown in Figure 2 for the vibrator source. Basic components are:

a. Pseudo-random noise generator.

b. Band-pass filters.

c. Power amplifier for vibrator.

d. Vibrator.

e. Geophones.

f. Cascaded amplifiers.

g. Tape recorder.

13. A description of these components follows:

a. Pseudo-random noise generator. A General Radio Company random noise generator, type 1390-B, with a frequency range of 5 Hz to 5 MHz was used as a broad band, uniform amplitude (white noise) signal source.

b. Band-pass filters. Krohn-hite model 3202R band-pass filters were used. These filters have two channels of input. One channel is operated in the low-pass mode and the other channel is operated in the high-pass mode to provide the desired noise bandwidth. The model 3202R will function as a band-pass filter with an attenuation rate of 24 db per octave outside the pass band. The
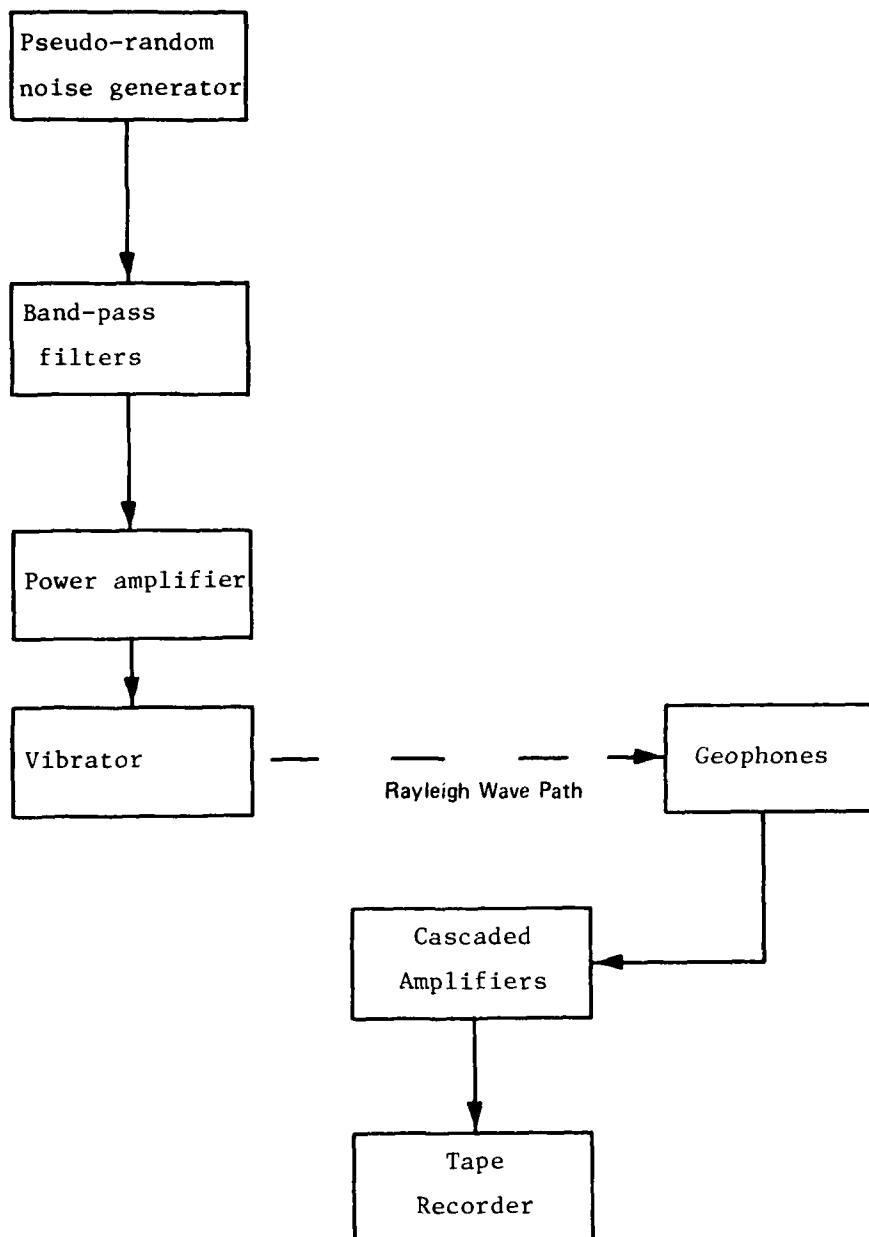
Figure 2. Equipment configuration for Rayleigh wave velocity measurements using vibrator source

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Krohn-hite filters have cutoff frequencies continuously adjustable over the frequency range from 20 Hz to 2 MHz.

c. <u>Power amplifier for vibrator.</u> A solid state Phase Linear Model 400 direct coupled amplifier was used to power the Goodman vibrator. The amplifier is capable of producing 700 W of signal power if driven to full output.

d. <u>Vibrator.</u> The Goodman vibrator (50 lb force*) was used as an energy source. The vibrator has a 16-ohm coil and has a power rating of 100 W.

e. <u>Geophones.</u> The analog signatures resulting from the energy source were measured with the Mark Products model L-1U geophones. They have a 4.5-Hz natural frequency and are commonly used in seismic refraction work. A total of four L-1U geophones were used. The amplitude and phase responses of the L-1U geophones are discussed in Appendix B of this report.

f. <u>Cascaded amplifiers.</u> Cascaded, common-mode rejection, operational amplifiers designed and constructed at the U. S. Army Engineer Waterways Experiment Station (WES) were used for amplifying the geophone signals. These amplifiers have adjustable gains from 0.1 to 1000.

g. <u>Tape recorder.</u> An FM magnetic tape recorder, Lockheed model 417, which records and reproduces seven channels of data at 1-7/8, 3-3/4, and 7-1/2 in./sec, giving a flat frequency response of direct current (DC) to 625 Hz, DC to 1250 Hz, and DC to 2500 Hz, respectively, was used for recording. The tape recorder also records in the direct mode. During these tests, all signatures were recorded at tape speeds of 7-1/2 in./sec and band limited from DC to 2500 Hz.

14. A photograph of the recording, monitoring, and amplifying instrumentation is presented as Figure 3. The L-1U geophone and Goodman vibrator are shown in Figures 4 and 5, respectively. Instrumentation identified in Figure 3, but not discussed as basic components of the data collection configuration, are:

a. Tektronix model 422 portable oscilloscope.

b. Continental Specialties Corporation electronic counter (100 MHz frequency counter) for alignment of the tape recorder.

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* A table of factors for converting U. S. customary units of measurement to metric (SI) is presented on page 3.
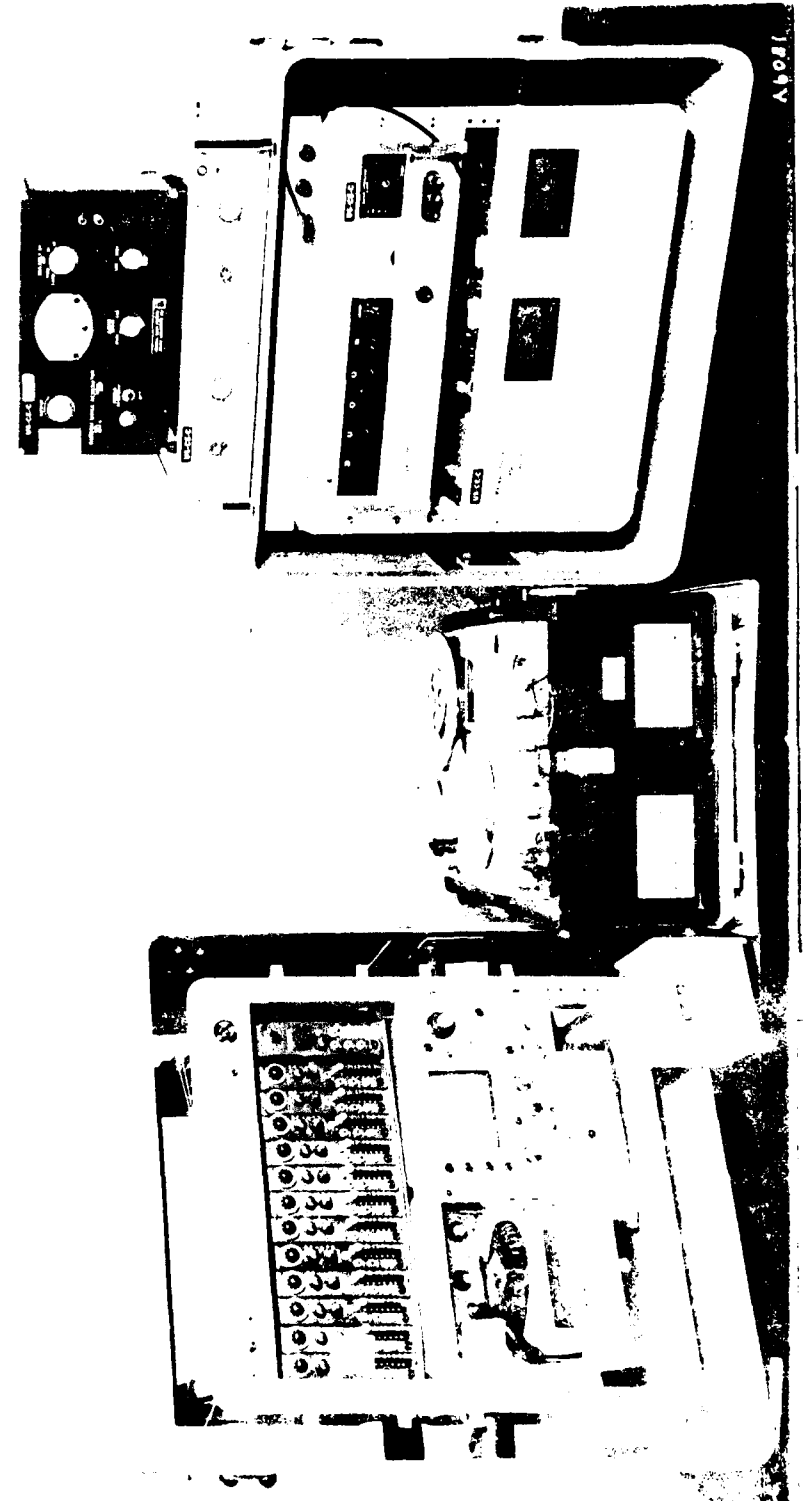
Figure 3. Recording, monitoring, and amplifying instrumentation.
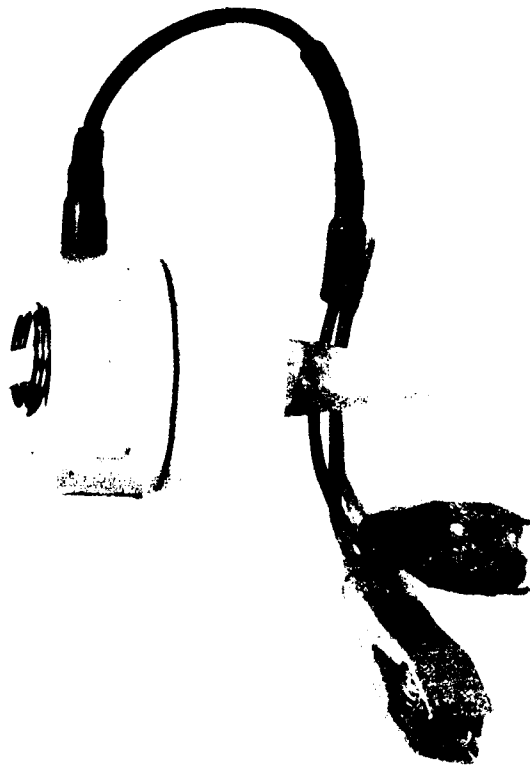
Figure 1. Tape Recorder ... [illegible]

Figure 2. Tape Recorder ... earphone

Test Sites

15. The general test area was located at WES in Vicksburg, Miss., on the northeastern most part of the installation. The in situ soils developed from loessial deposits. The surface soil was a brown to dark brown heavy silt loam (CL) and the subsoil ranged from heavy silt loam (CL) to silty clay loam (CL). The in situ medium consisting of this loess was selected for one of the two test sites identified in paragraph 11. Previous measurements of shear wave velocity of the loess medium at the first site were on the order of 120 m/sec within several metres of the surface. The measured compression wave velocity in the upper two metres was about 350 m/sec. The second site (soil-cement site) in the test area contained a mixture of loess, sand, and cement placed in deep loess soil in a pit 6.0 m wide, 6.5 m long, and 0.92 m deep. The mixture was constructed to have shear and compression wave velocities of approximately 600 and 1000 m/sec, respectively. The soil-cement test site is further described in Appendix C.

Data Collection Procedures

16. The procedures for data collection consisted of the following:

a. Equipment setup. Recording instrumentation and source and geophones were configured for data collection. Geophones were placed radially outward from the source (vibrator or impulse derived from a hammer drop). The relative locations of the source and geophones are identified in Figure 6.

b. Source activation. For the vibrator, the driving frequencies were band passed through the filters identified previously. The frequency bands were: 25 to 100, 100 to 200, 200 to 300, 300 to 400, 400 to 500, 500 to 700, 700 to 1200, 1200 to 2400, and 25 to 2400 Hz. Once amplifier gains were adjusted appropriately for tape recording, 30 sec of vibrator-induced data was recorded. Immediately following the vibrator source tests, three successive impulses from the hammer drop were recorded.
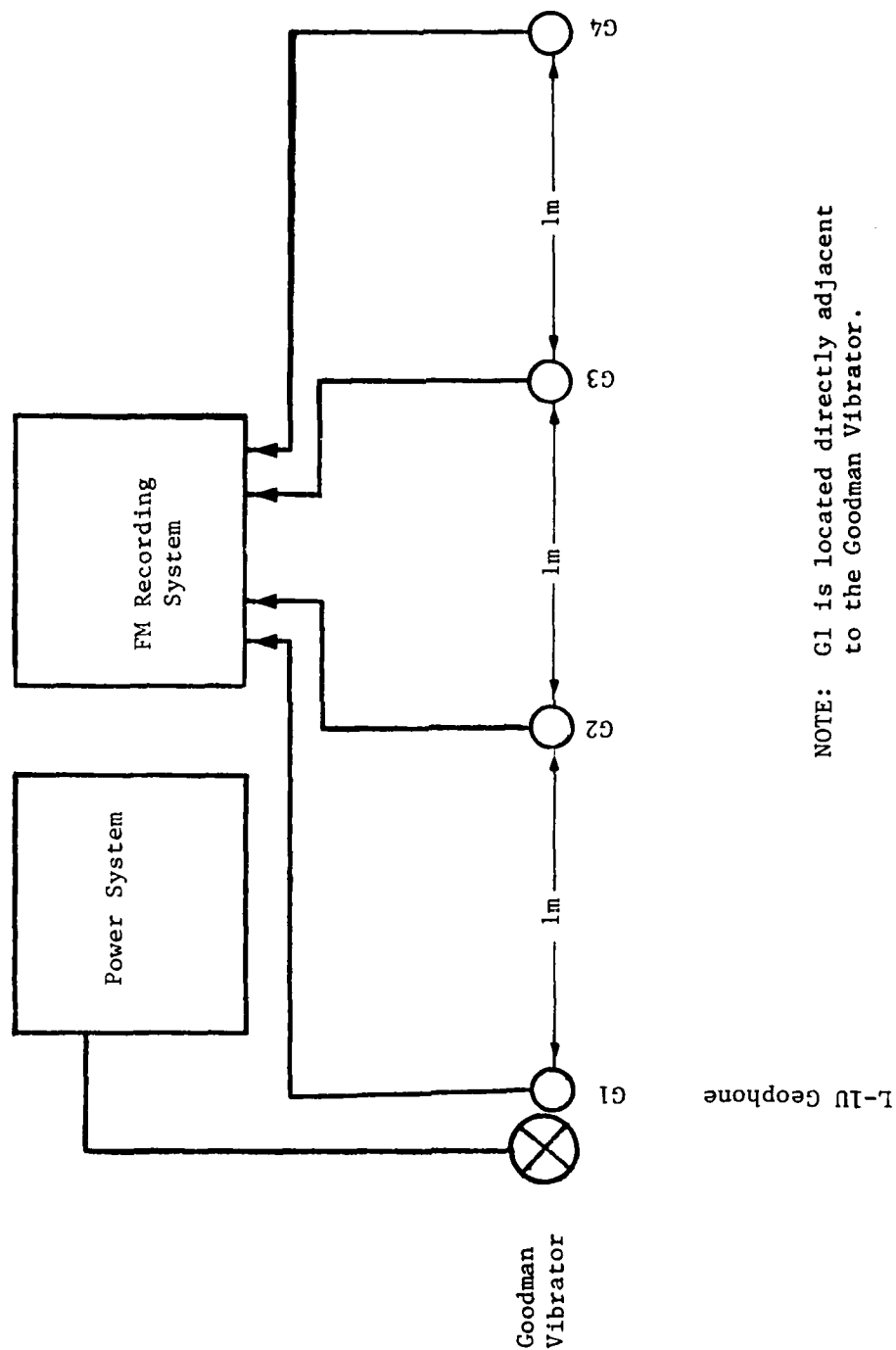
NOTE: G1 is located directly adjacent to the Goodman Vibrator.

Figure 6. Layout of equipment on test site

PART IV: DATA ANALYSIS

Introduction

17. The data analysis procedures were developed to emphasize the
following points:

    a. Demonstrate the measurement of the Rayleigh wave velocity
for the usual frequency region for such measurements (20
to approximately 400 Hz).

    b. Evaluate the potential for extending such measurements to
frequencies higher than 400 Hz and identify limitations
in such an extension.

    c. Define alternative instrumentation, data collection pro-
cedures, and analysis techniques that might be employed
to improve and expedite field measurement of Rayleigh
wave velocities using broad band sources.

18. The results of the data analysis procedures are dependent on
the methods used for processing the geophone signatures. The operations
involved in such an analysis consisted of:

    a. Digitizing the recorded signatures.

    b. Computing the Fourier transform and cross-spectrum
properties of the signatures from any two geophone
signatures.

The methods used for cross-spectrum analysis to compute the needed phase
difference (Equation 7) are discussed in Appendix A. A particularly
useful function for evaluation of the accuracy of the results is the
squared coherence (Equation A3, Appendix A). The squared coherence has
values ranging from 0 to 1. Values of squared coherence near unity are
observed when the phase differences between two signatures in a narrow
frequency band (the band width depends on the stability of the signa-
ture characteristics over time) are consistent for repeated samples
selected from the signature record. Therefore, the coherence between
the geophone signatures was computed to provide at least a qualitative
evaluation of the accuracy of the estimate of the phase difference.

19. The results reported herein were obtained using the following
analysis procedure:

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a. Analog data were digitized at a rate of 5000 samples per second.

b. A 4096-point Fourier transform was computed (for a 0.8-sec time window).

c. Spectral parameters (G_x, G_y, G_{xy} in Appendix A) were averaged over four successive time windows (each of 0.8-sec duration).

Although data were recorded for a number of frequency bands (paragraph 16b), the results were very similar for successive bands. Therefore, only the extreme conditions represented by the lowest band (25 to 100 Hz) and the entire band (25 to 2400 Hz) are presented for discussion.

Analysis Results

Vibrator source

20. Measured phase differences between the signatures for geophones 1 and 2 for the loess site using the 25- to 100-Hz random noise signal to drive the vibrator are shown in Figure 7a. The phase differences "flip" from $+\pi$ to $-\pi$ at about 50 Hz and again at approximately 135 Hz. Although the driving frequencies were filtered outside the 25- to 100-Hz bank, sufficient energy existed in the driving signals to provide continuous phase difference information to well above 100 Hz. The squared coherence for these data is presented in Figure 7b. When the squared coherence is continuously greater than approximately 0.8 over a bandwidth exceeding 10 Hz, it can generally be regarded as "good" in the sense that the phase difference measurements are very stable from one sampled frequency to the next. The coherence is quite good from about 20 Hz to approximately 300 Hz. It also has apparently good coherence below 10 Hz. However, the "good" appearance is largely an artifact of the logarithmic scale for which the first cycle (1 to 10 Hz) occupies a third of the total plot so that the high coherence levels are only several Hertz wide. The resulting velocities (derived using Equation 9 for which the total phase difference is used by including the factor of 2π increase for each flip) as a function of frequency are presented in Figure 7c. Figure 7c shows that there is some disperison over the
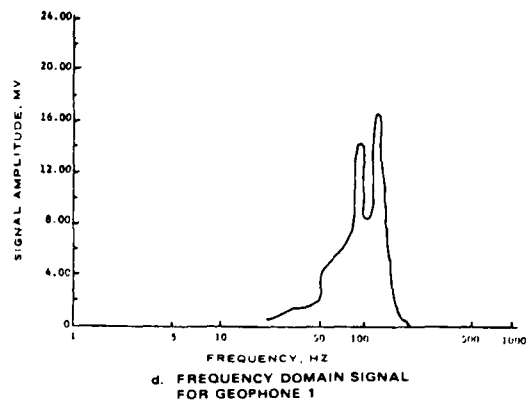
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a. PHASE DIFFERENCE

b. COHERENCE

c. RAYLEIGH WAVE VELOCITY

d. FREQUENCY DOMAIN SIGNAL FOR GEOPHONE 1
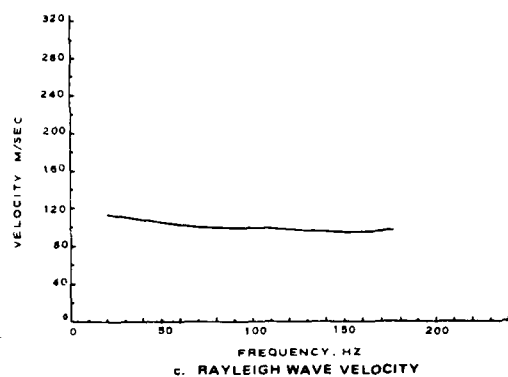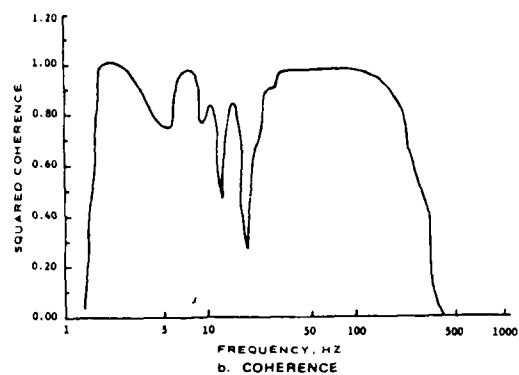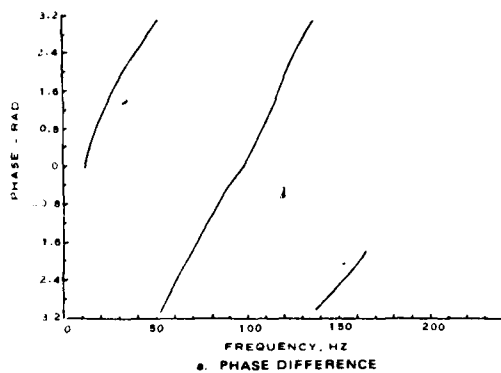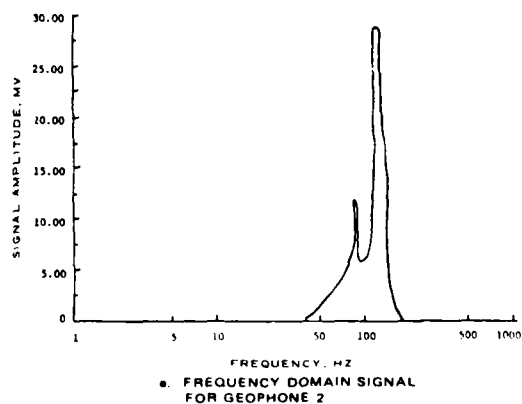
e. FREQUENCY DOMAIN SIGNAL FOR GEOPHONE 2

Figure 7. Results for soil (Vicksburg loess), 25 to 100 Hz, geophones 1 and 2

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frequency region 20 to 160 Hz. The measured velocity is about 120 m/sec at 20 Hz and drops to about 100 m/sec between 100 and 160 Hz. These measurements can be compared with those tabulated below, which were obtained on the same soil type during another data collection effort using the single frequency (conventional) Rayleigh wave velocity measurement technique discussed in paragraph 3.

Soil Medium	Frequency, Hz	Rayleigh Wave Velocity, m/sec
Soil (loess)	100	125
	150	150
	200	125
	250	125

The latter measurements are somewhat higher than those observed using the broad band noise source but still show reasonable agreement. The Rayleigh wave velocities were obtained on somewhat different soil conditions (i.e. the soil moisture content for the two measurements was not identical) and the differences (i.e. approximately 25 percent) are considered small relative to their range for naturally occurring conditions (i.e. comparing results for frozen and nonfrozen conditions for soils or for soil and rock).

21. The spectrum of the signal induced by the vibrator driven in the frequency region 25 to 100 Hz is presented in Figure 7d and 7e for geophones 1 and 2, respectively. The amplitude of the induced vibrations is predominantly in the region 70 to about 150 Hz. The reasons for the low amplitude in the 25- to 50-Hz region are: (a) low force levels output from the vibrator, and (b) poorer coupling of forces to induce vibrations at the lower frequencies relative to the coupling at higher frequencies.

22. Results of the phase difference calculations for the band 25 to 2400 Hz are presented in Figure 8a for geophones 2 and 3 for instances in which the squared coherence exceeds 0.8. The squared coherence plot for this band is presented in Figure 8b. The squared coherence is generally good (exceeds 0.8) across the range 20 to 450 Hz
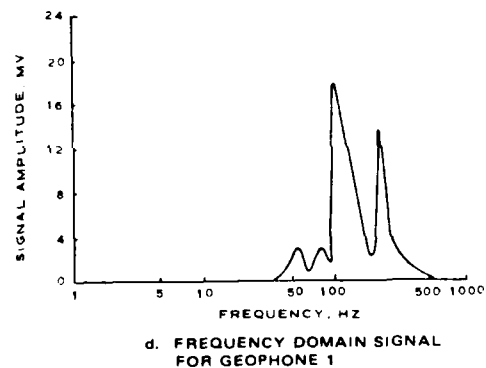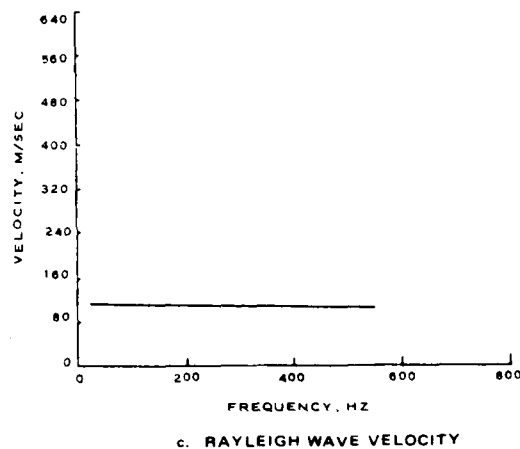
a. PHASE DIFFERENCE

b. COHERENCE

c. RAYLEIGH WAVE VELOCITY

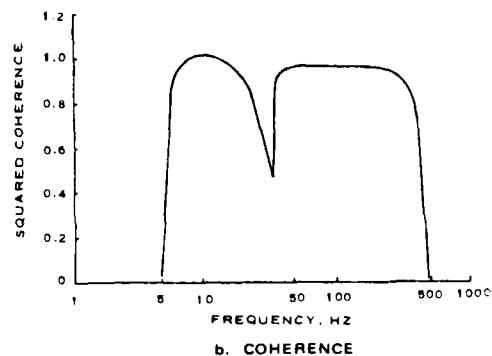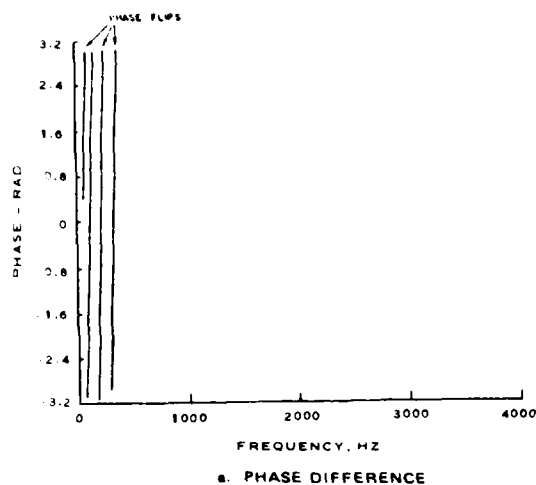d. FREQUENCY DOMAIN SIGNAL
FOR GEOPHONE 1

Figure 8. Results for soil
(Vicksburg loess), 25 to
2400 Hz, geophones 1 and 2

e. FREQUENCY DOMAIN SIGNAL
FOR GEOPHONE 2

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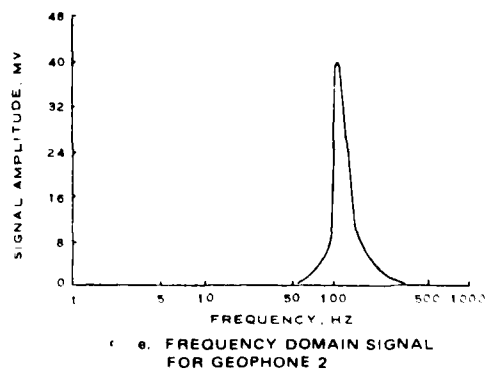
except in the frequency region 30 to 40 Hz. The resulting velocities
for the frequency region 0 to 500 Hz (for which the squared coherence
exceeds 0.8) are presented in Figure 8c. Although the data are not pre-
sented herein, for frequencies above 500 Hz, the squared coherence is
so poor that the computed velocities are highly erratic and do not re-
sult in a continuous curve of velocity as a function of frequency. This
can be attributed to the low amplitude of the signals at these high fre-
quencies and to the erratic phase response of the geophones (see Figure
B1 in Appendix B). The low amplitude of the signals at the high fre-
quency (see Figures 8d and 8e) is primarily due to the attentuation of
the propagating waves in the soil (due to viscus damping). The phase
response of the geophones is discussed further in Appendix B.

23. Results of velocity computations using various geophone com-
binations (1 and 2, 2 and 3, 3 and 4) on the loess are presented in Fig-
ure 9a. The velocity computations were made using the 25- to 2400-Hz
broad band noise source. The dependence of velocity on frequency is
about the same for each pair of signals used in the computation. How-
ever, there is a 20 to 30 percent difference in the calculated veloc-
ities for the maximum computed velocity (geophones 2 and 3 or 3 and 4)
and the minimum computed velocity (usually geophones 1 and 2). Differ-
ences in velocity derived from different pairs of geophones may be
attributable to: (a) geometric dependent mixing of Rayleigh wave motion
induced by the source with other motions (such as background noise com-
ing from a direction different from that of the source, reflections of
compression and shear waves off of deeper layers, etc.); (b) differences
in the phase response of the geophones used for recording; and (c) non-
homogeneities in the media between the pairs of geophones. To the ex-
tent that b and c contribute to differences among geophone pairs, the
most representative velocity is probably best determined by averaging
the three velocity curves. (Similar comparisons of measurements from
geophones 1, 2, 3, and 4 were made for the soil-cement site and the
results are shown on Figure 9b; also see comments about results at
frequencies above 500 Hz in paragraph 25.)

24. The phase differences derived from geophones 1 and 2 on the

22
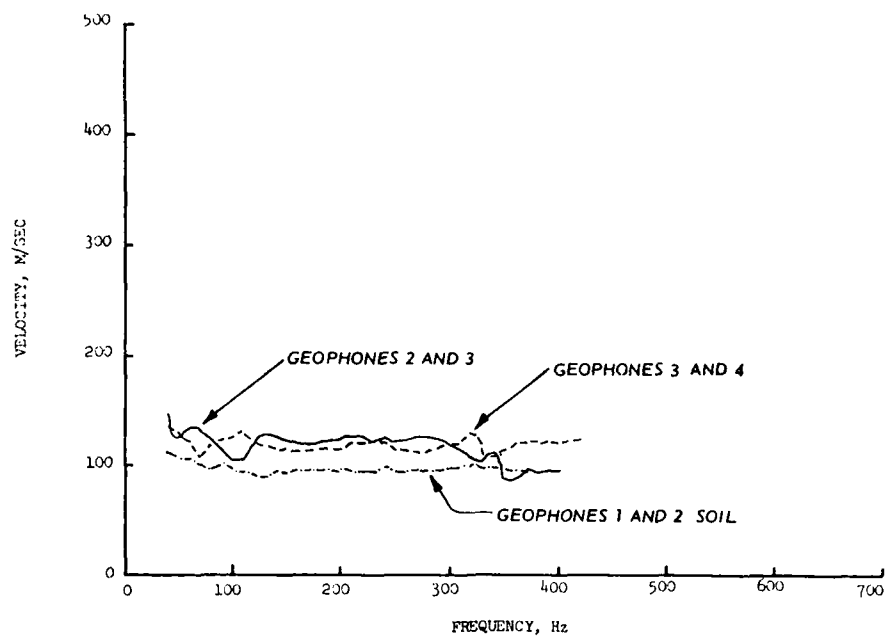
a. Velocity for Vicksburg loess
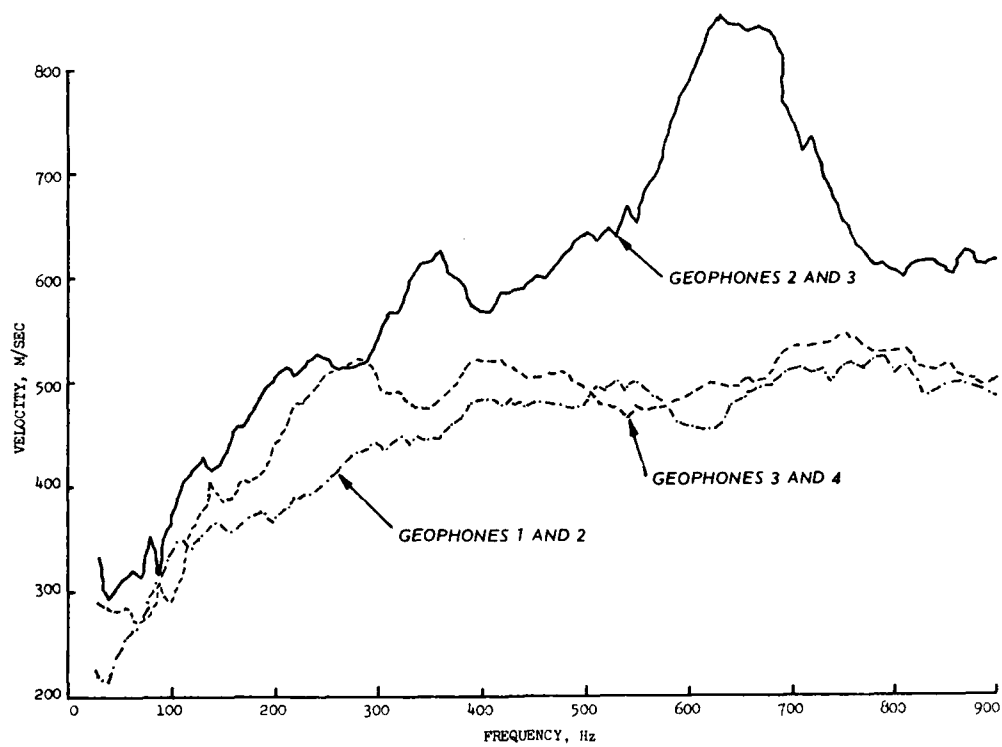
b. Velocity for soil cement

Figure 9.   Effect of geophone combination for measurements
in loess and soil cement

soil-cement site for the 25- to 100-Hz source-driving frequencies are
presented in Figure 10a. No phase flips occur over the frequency range
20 to 170 Hz. As shown in Figure 10b, the coherence is good over the
frequency range tested and in fact good to 290 Hz. Since the distance
between geophones 1 and 2 is the same for the soil-cement site as for
the loess site, the velocity in the soil cement is considerably greater
than in the loess as evidenced by the phase flips for the loess (Fig-
ure 7a). The resulting measurements of the Rayleigh wave velocity over
the 20- to 170-Hz frequency range are presented in Figure 10c. Again,
the low amplitude at the higher frequencies (Figure 10d and 10e) is due
primarily to filtering out the driving frequencies to the vibrator
above 170 Hz.

25. The phase differences derived from the 25- to 2400-Hz source-
driving frequencies are presented in Figure 11a. Phase flips are ap-
parent at approximately 200 and 700 Hz. Both the phase difference plot
and the squared coherence plot (Figure 11b) show a continuous nature to
frequencies as high as 1000 Hz. The presence of consistent phase dif-
ference information to such high frequencies as apparent in the presen-
tation of the squared coherence (Figure 11b) can be best accounted for
by the fact that high frequencies are attenuated less severely with
distance in the rigid soil-cement medium than in the loess medium.
Therefore, the signal levels of the propagating waves are well above
the recording noise level at the higher frequencies for the soil-cement
medium. The velocities resulting from the phase difference computation
for the 20- to 2400-Hz band are presented in Figure 11c. The curve
shows the velocity increasing as the frequency increases with the
velocity at high frequencies approaching 500 m/sec. However, as shown
in Appendix B, the difference in phase responses of L-1U geophones be-
comes questionable at frequencies over 500 Hz. This point is also
demonstrated in Figure 9b, as the velocity plots above 550 Hz are some-
what erratic. An alternate transducer, such as an accelerometer, would
be better suited for making velocity measurements at such frequencies.
Nevertheless, the clear interpretation of the phase flips over a broad
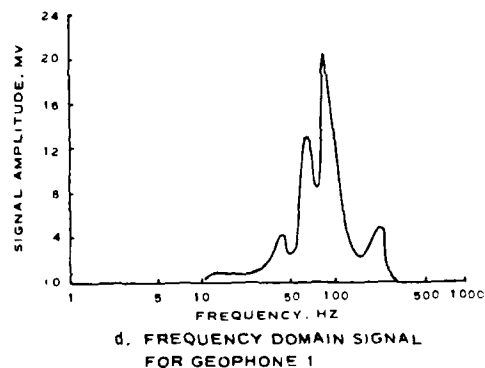range of frequencies (see Figures 11d and 11e) demonstrates the

a. PHASE DIFFERENCE

b. COHERENCE

c. RAYLEIGH WAVE VELOCITY

d. FREQUENCY DOMAIN SIGNAL
FOR GEOPHONE 1

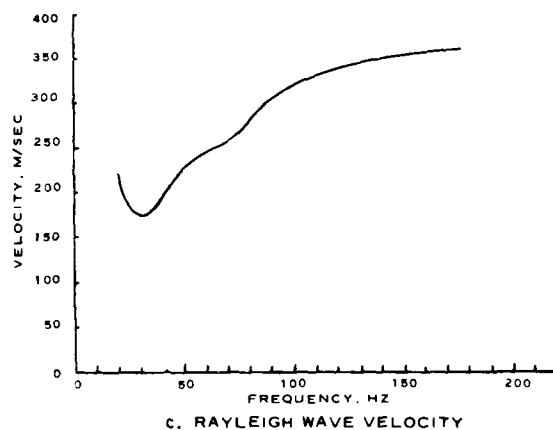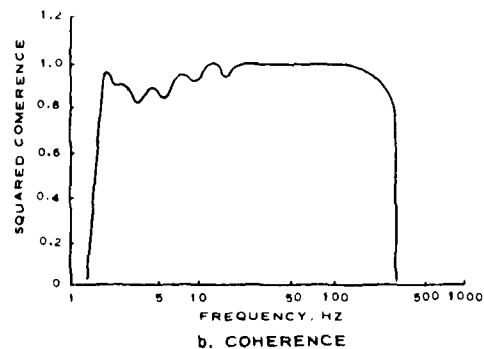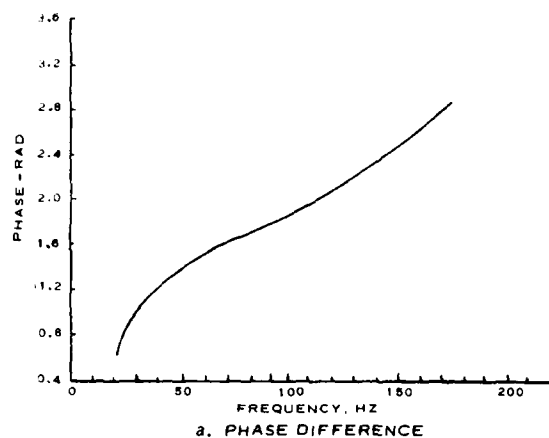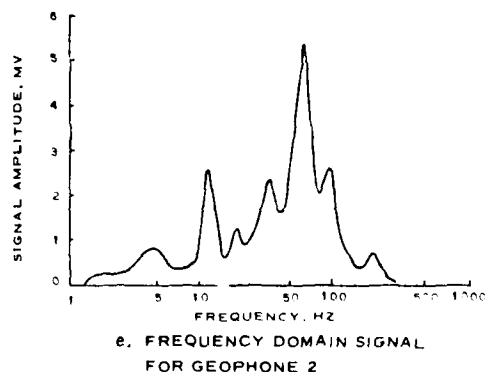e. FREQUENCY DOMAIN SIGNAL
FOR GEOPHONE 2

Figure 10. Results for soil cement,
25 to 100 Hz, geophones 1 and 2

25

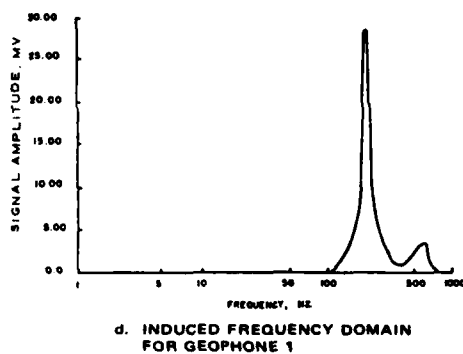a. PHASE DIFFERENCE

b. COHERENCE

c. RAYLEIGH WAVE VELOCITY

d. INDUCED FREQUENCY DOMAIN
FOR GEOPHONE 1

Figure 11. Results for soil cement, 25 to 2400 Hz, geophones 1 and 2

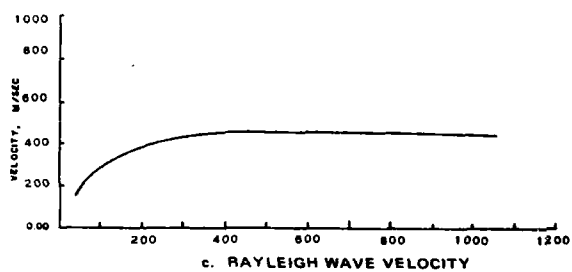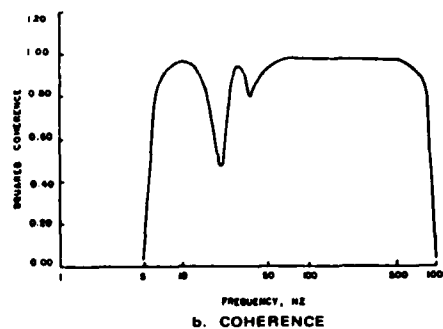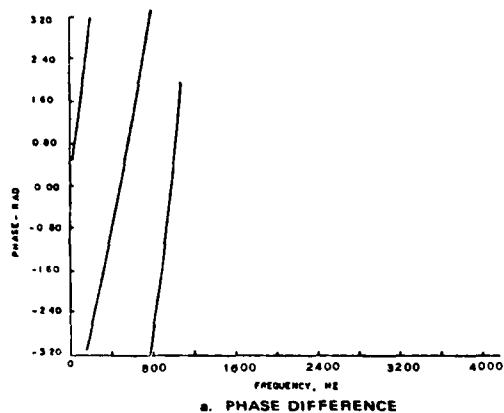e. INDUCED FREQUENCY DOMAIN
FOR GEOPHONE 2

26
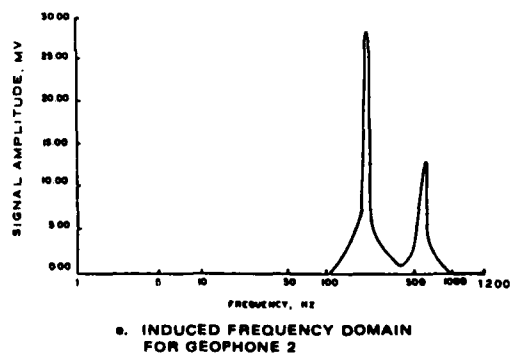
advantages of the broad band source and associated signal processing for deriving the Rayleigh wave velocity, particularly at high frequencies. Measurements taken on the same soil type (soil cement) using the conventional single frequency Rayleigh wave velocity measurement technique are tabulated below:

Soil Medium	Frequency, Hz	Rayleigh Wave Velocity, m/sec
Soil cement	250	360
	300	375
	350	375
	400	375

Although data at frequencies above 400 Hz for the conventional single frequency Rayleigh wave velocity measurement technique are not presented here, a comparison of the data from both techniques clearly indicates that the broad band source technique has potential for providing more consistent results at the higher frequencies.

Impulse source

26. In field implementation of measurement of Rayleigh wave velocities using broad band sources, use of an impulse source, such as a hammer drop, would simplify test requirements and would be particularly expedient for military application. This section of the report presents the results of processing the geophone signals derived from hammer drops to obtain the Rayleigh wave velocity. Figure 12a presents the computed phase differences derived from a single hammer drop 2 m from geophone 1 for the loess site. The phase difference curve is erratic in the 55- to 70-Hz region but has fairly good continuity in the frequency regions 20 to 55 and 70 to 100 Hz. Because of the short duration of the impulse, it is difficult to get an accurate measure of the squared coherence. However, the computed squared coherence is presented in Figure 12b and shows a definite drop in the frequency region of 55 to 70 Hz.

27. The Rayleigh wave velocity computed from the hammer drop is presented in Figure 12c. In the 55- to 70-Hz region, the velocity deviates dramatically from 20 to 220 m/sec. If the 55- to 70-Hz frequency region is overlooked, the measured Rayleigh wave velocity derived from the impulse agrees reasonably well with that observed for the loess

27

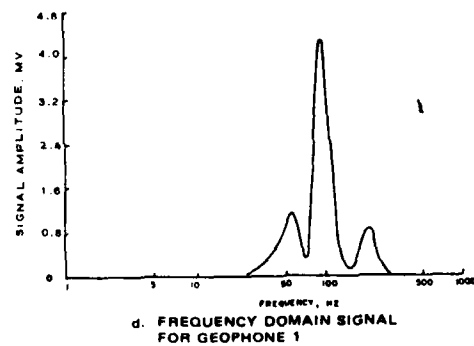a. PHASE DIFFERENCE

b. COHERENCE

c. RAYLEIGH WAVE VELOCITY

d. FREQUENCY DOMAIN SIGNAL FOR GEOPHONE 1
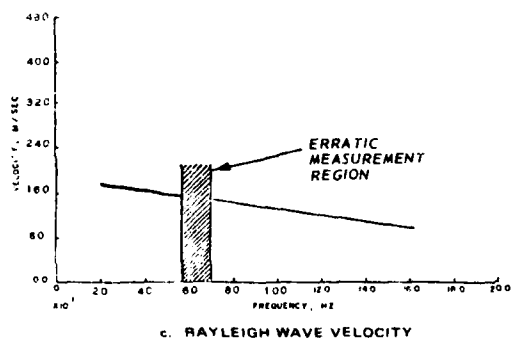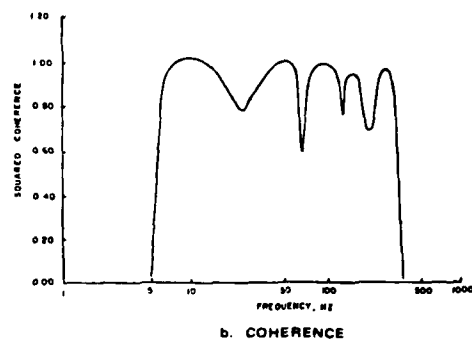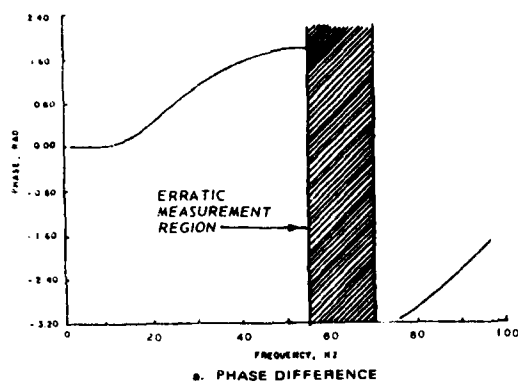
e. FREQUENCY DOMAIN SIGNAL FOR GEOPHONE 2

Figure 12. Data results produced by impulse (hammer) on soil (Vicksburg loess), geophones 1 and 2

28
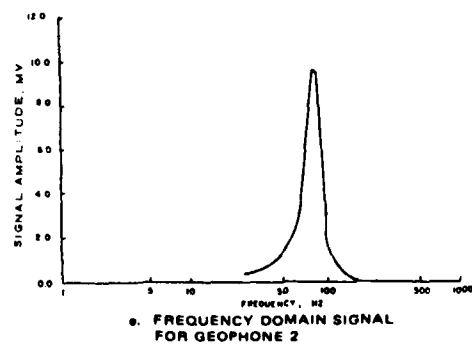
using the broad band vibrator source (compare Figure 9a with Figure 12c).
The measured velocities for the impulse source are somewhat higher in
the frequency bands 20 to 55 Hz than for the vibrator results but agree
very well over the 70- to 160-Hz frequency region. The lack of coher-
ence in the 55- to 70-Hz frequency region may be related to the dip in
the amplitude spectrum in this region (Figures 12d and 12e). Improve-
ment in the velocity accuracy over the 20- to 55-Hz band could probably
be obtained by increasing the signal amplitude at these frequencies.
However, because of its short duration, a single impulse does not pro-
vide much statistical opportunity for averaging cross-spectral param-
eters (Appendix A). Impulses may provide adequate information for
deriving the Rayleigh wave velocity curve provided repeated impulses
are incorporated into an averaging procedure.

PART V: CONCLUSIONS AND RECOMMENDATIONS

Conclusions

28. Based upon the study reported herein, the following conclusions can be drawn:

    a. The limitations in measuring Rayleigh wave velocities using conventional monofrequency techniques at frequencies higher than 200 Hz can be circumvented by using a multifrequency (broad band) source and special signal processing techniques (paragraphs 7 and 19). Processing of geophone signatures by computation of the Fourier transform and cross-spectrum properties (phase difference and squared coherence) will indicate where velocity measurements are reliable and where they are questionable.

    b. The results of the impulse source method, described herein, compare very favorably with the vibrator source that generates frequencies over a broad region. Impulses may provide adequate information for deriving the Rayleigh wave velocity curve provided repeated impulses are incorporated into an averaging procedure (paragraph 27).

    c. Rayleigh wave velocity measurements of frequencies above 600 Hz are for the most part limited by the measurement transducers (paragraph 25). The accuracy of the procedure described herein is limited only by the instrumentation and equipment used (especially the phase response of the geophones).

Recommendations

29. It is recommended that:

    a. Alternate transducers that will more reliably measure high frequency signals (exceeding several hundred Hertz) be used (i.e. accelerometers) in place of geophones.

    b. More extensive tests be conducted to improve the accuracy using the impulse source technique where repeated impulses are incorporated into an averaging procedure.

    c. Commercially available spectrum analyzers, capable of computing the Fourier transform and spectral properties of multifrequency signals within seconds after the data have been measured, be incorporated and used along with the procedures described in this study to compute phase velocity for Rayleigh waves propagating in terrain materials.

30

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Stabler, J. R., Baylot, E. A., and Cress, D. H. 1976. "Frequency Dependent Wave Arrival Time Delays in Dispersive and Nondispersive Media," ARO Report 76-1, Transactions of the Twenty-First Conference of Army Mathematicians, El Paso, Tex.

APPENDIX A: MEASUREMENT OF SPECTRAL PROPERTIES

1.   This appendix discusses the measurement of phase difference, wave velocity, and the squared coherence of signals using Fast Fourier Transform (FFT) techniques.

2.   The geometric relationship addressed in this discussion is depicted in Figure A1.   As shown, it consists of sensors spatially

**PLANE
WAVES**

**ENERGY
SOURCE**   d   **+Z DIRECTION**

                                                              **SENSOR   SENSOR
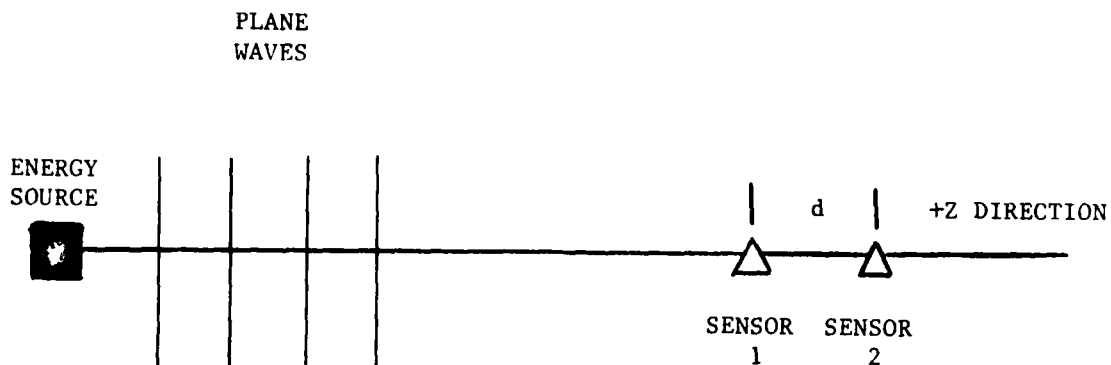   1   2**

Figure A1.   Geometric relation between sensors and incoming plane waves

separated by a soil media (soil/soil cement).   The energy source (Goodman vibrator) produces a series of waves that are assumed to be propagating away from the source along the line in which the geophones are placed.   For the purpose of this study, it is assumed that a single mode of propagation is employed.   This avoids the ambiguity that may occur in estimating wave velocities when several vibrational modes having different propagational velocities are present, hence, leading to different velocity measurements for each mode of vibration (Stabler, Baylot, and Cress 1976).

3.   When energy is input into the soil, as in this situation, it is spread over bands encompassing a broad range of frequencies.   When a broad region of frequencies is present in the signatures, a plot of phase difference versus frequency can be generated.   It is important at this point to recognize that the speed (velocity) at which a particular wave travels from one transducer to the next is directly related to the phase difference ϕ .   As discussed by Stabler, Baylot, and Cress (1976) mathematically,

A1

$$\phi = kd \qquad\qquad (A1)$$

where

      k = wave number = $2\pi f/v$

      d = point along line in which waves are propagating

      f = frequency

      v = velocity

    f/v = wavelength

then

$$\phi = 2\pi fd/v$$

To eventually find the velocity v of the wave, the quantity is given by

$$v = \frac{2\pi fd}{\phi} \qquad\qquad (A2)$$

4. The process of computing the phase differences for each discrete frequency band in the frequency domain has been made easier with the advent of the FFT. It is now practical to compute the whole spectrum in reasonable computation time. Even with this computation technique (FFT) of the phase difference, some applications of this approach may produce inconsistent results (Stabler, Baylot, and Cress 1976). For example, the calculated phase difference between two signatures can be so erratic in some frequency bands that it is impossible to identify which estimates, if any, are reliable. As discussed by Hammon and Hannan (1974), the coherence parameter γ_{xy}^2 can be directly related to the accuracy of the cross-spectral estimate (hence phase difference) at a particular frequency. Because the accuracy of the phase difference estimate improves rapidly as the coherence increases toward unity, the coherence parameter can be used to determine the frequencies that can be used to provide the best estimate of phase difference, thereby, providing improved estimates of wave velocities. The squared coherence is computed using the results of the FFT. In the FFT, the spectral content of a time series is found using (Flohr and Cress 1979)

A2

$$X_k = \Delta t \sum_{m=0}^{N-1} x_m \; \exp\left(-i2\pi f_k m \Delta t\right) \qquad (A3)$$

where

$$X_k = \text{complex amplitude associated with frequency } f_k$$
$$\text{Real } (X_k) = \text{amplitude of cosine transform at frequency } f_k$$
$$\text{Imaginary } (X_k) = \text{amplitude of sine transform at frequency } f_k$$
$$\Delta t = \text{time between samples}$$
$$N = \text{total number of samples}$$
$$x_m = \text{amplitude of sampled function at time} = m\Delta t$$
$$i = \sqrt{-1}$$
$$f_k = k/N\Delta t$$
$$k = 1 \ldots N$$

Then the auto spectrum is denoted by

$$G_{xk} = X_k {}^* X_k \qquad (A4)$$

Where the astrisk denotes complex conjugation.. The cross spectrum
for two time series x(t) and y(t) is given by

$$G_{xyk} = X_k {}^* Y_k. \qquad (A5)$$

Then the coherence parameter γ_{xy}^2 is given by

$$\gamma_{xyk}^2 = \frac{\left| \hat{G}_{xyk} \right|^2}{\hat{G}_{xk} \hat{G}_{yk}} \qquad (A6)$$

where the circumflex (^) indicates an average value. For our analysis,
the averaging was done over adjacent frequency values using a centrally
weighted weighting function. For example

$$\hat{G}_{xyk} = \frac{1}{M} \sum_{j=1}^{7} G_{x(k+j-4)} W_j \qquad (A7)$$

A3

where $W_j = 1,2,4,6,4,2,1$ for $j = 1,2,3,4,5,6,7$, respectively, and

$$M = \sum_{j=1}^{7} W_j = 20 \qquad \text{(A8)}$$

A4

1. The purpose of this appendix is to present data from tests performed to verify the accuracy and reliability of the L-1U geophones used in this study. The verification is based on manufacturer specification. The L-1U geophones are produced by Mark Products, Houston, Tex., and have a natural frequency of 4.5 Hz. They are commonly used in seismic refraction work. Figure 4 of the main text is a picture of an L-1U geophone. A total of four L-1U geophones were used.

2. The conventional methods of verifying geophone reliability or accuracy are by (a) using a vibration pickup shake table to sweep through the desired frequencies at a constant velocity and plotting the phase and amplitude of a single geophone, or (b) checking the phase difference between geophones using the impulse source technique described in paragraph 26 of the main text. These methods are described in detail in the paragraphs below.

Shake Table Method

3. Each L-1U geophone was individually placed on a MB electronics Hamden, Conn., type 120, vibration pickup shake table. This particular table was designed to have axial resonance above 2 KHz, which is well above the working range of the pickup to eliminate any effect on the operation of the geophone. Each geophone was tested at a constant velocity (0.1 in./sec) and at the frequencies and sweep rates listed below:

Frequency, Hz	Sweep Rate, decade/min
2-10	1
10-100	1-0.2*
100-200	0.2
200-1000	0.1

* Sweep rate was decreased by 0.1 decade/min for each 10-Hz frequency increase

During each test, a tracking filter band width of 1 Hz was used.

4. The results of the shake table tests are shown in Figures B1 and B2. Figure B1 (a thru d) indicates that the geophones were reasonably close to manufacturer specifications. For each plot of the phase, the curve passes through 90 deg at the natural frequency (4.5 Hz) of the L-1U geophone. The phase plots (Figure B1) also indicate that the performance of the geophones was "good" up to approximately 195 Hz. Each geophone, except geophone 1, indicates consistent performance for approximately 195 Hz to 700 Hz. Geophone 1 shows consistency from approximately 195 Hz to 520 Hz. The amplitude plots for each geophone (Figure B2 (a thru d)) show an initial peak of about 0.60 V at approximately 5 Hz, which is appropriate for an underdamped geophone at the stated natural frequency of the geophones. Other erratic peaks occur at approximately the same high frequency ranges listed for the phase plots in Figure B1. The upper limit for reliable consistency also matches that of Figure B1. This test also indicates that the data analysis and conclusions in the main text were reasonably correct.

Impulse Source Method

5. As stated earlier, the steps of this test are described in paragraph 26 of the main text, the only difference being that the signals from the hammer drop are analyzed with respect to the phase difference of two geophones rather than from the source to one geophone. The phase difference was computed and plotted (Figure B3) for comparison of geophones 1 and 2, (Figure B3a), geophone 1 and 3, (Figure B3b), and geophones 2 and 3 (Figure B3c). Data from geophone 4 were incoherent for this test and were not included in this analysis. In Figures B3a and B3b, the phase difference stays near 0 rad to approximately 550 Hz where a significant roll-off is indicated. In Figure B3c, the phase difference stays near 0 rad to approximately 625 Hz before any roll-off occurs.

Conclusions

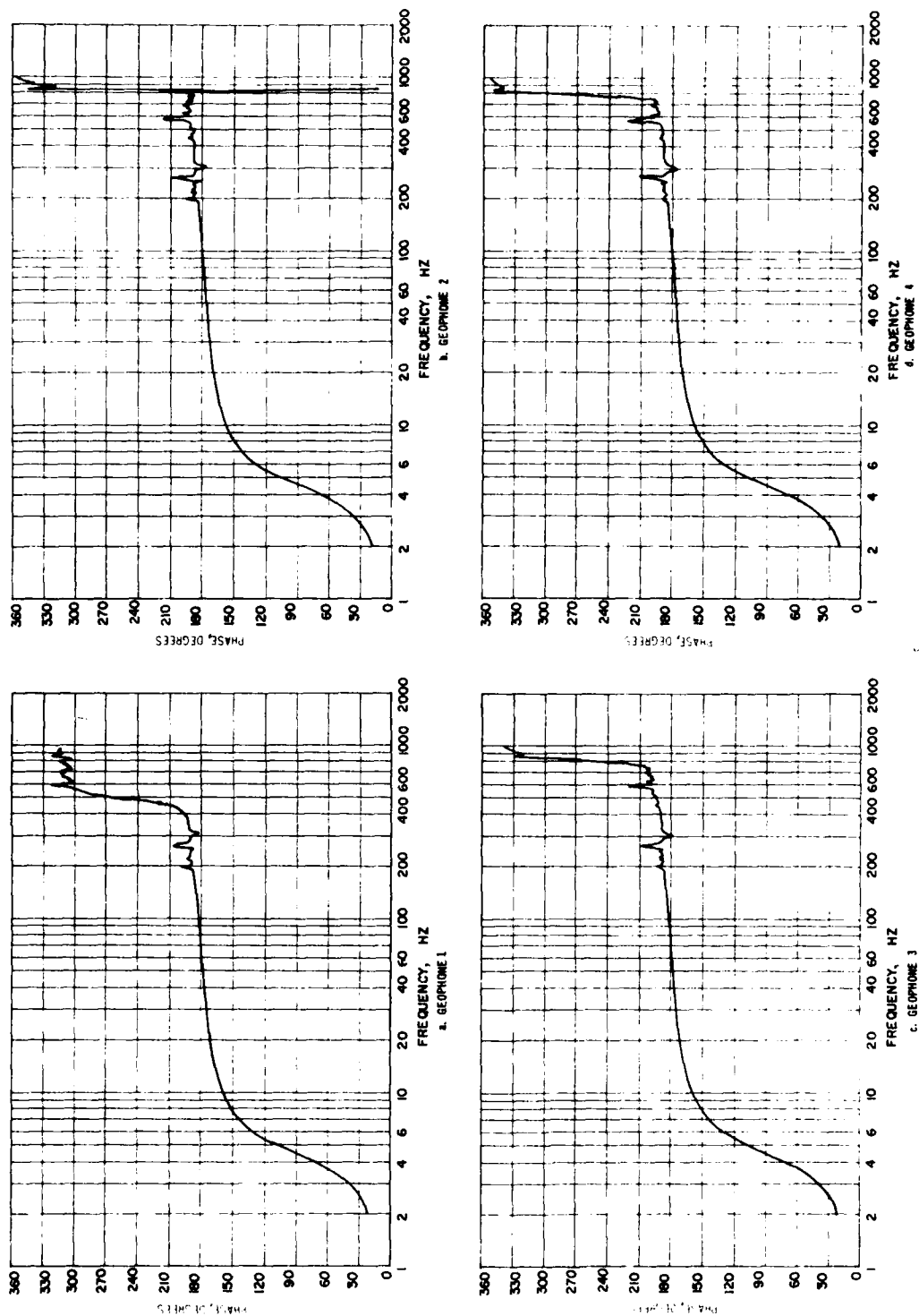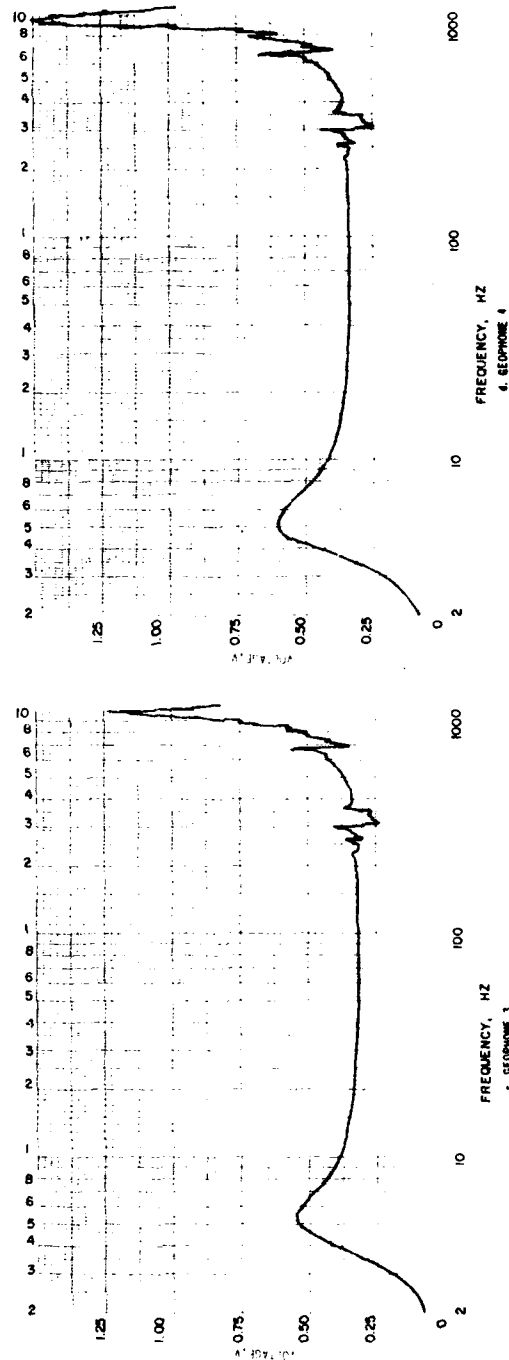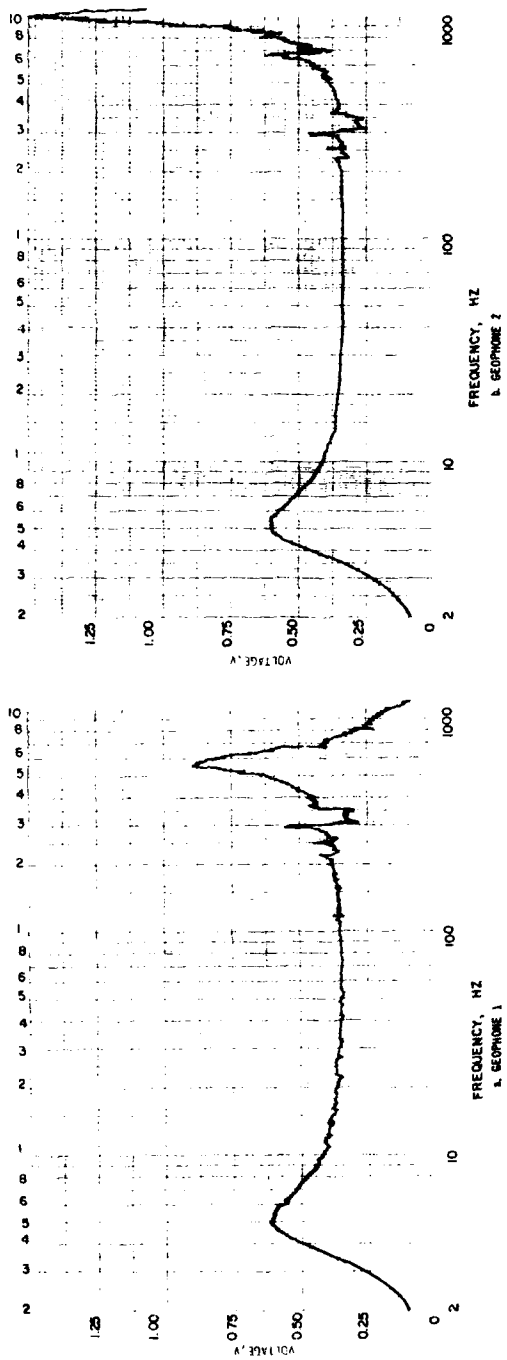6. As a result of these test analyses, it can be concluded that

B2

Figure B1. Phase plots on shake table

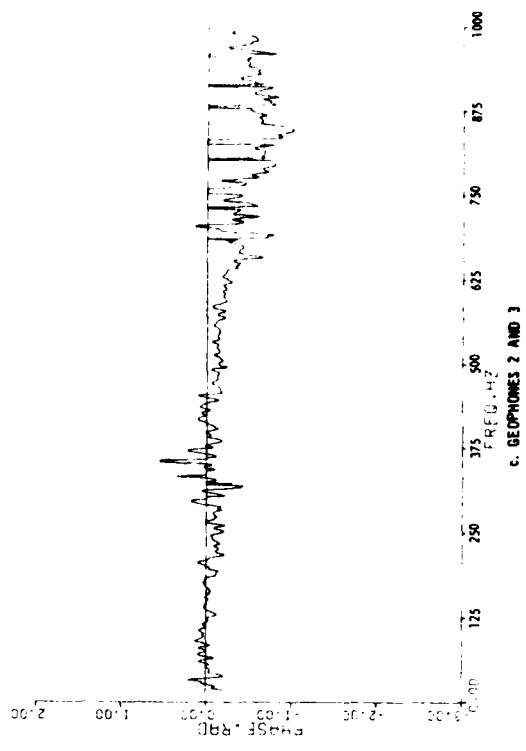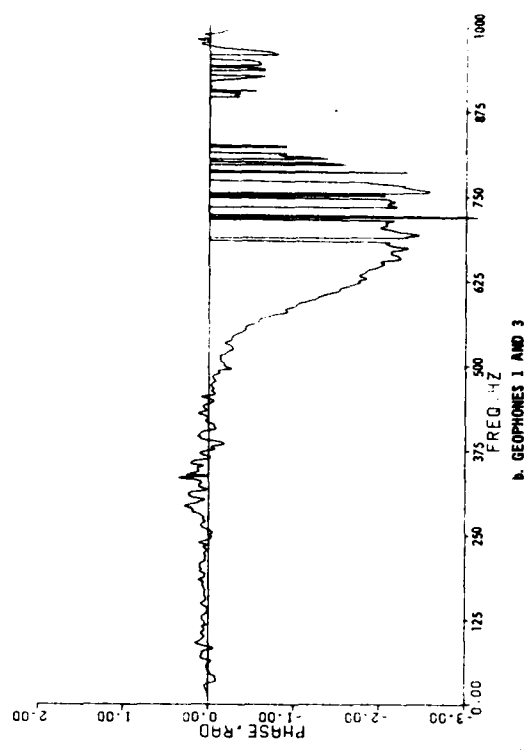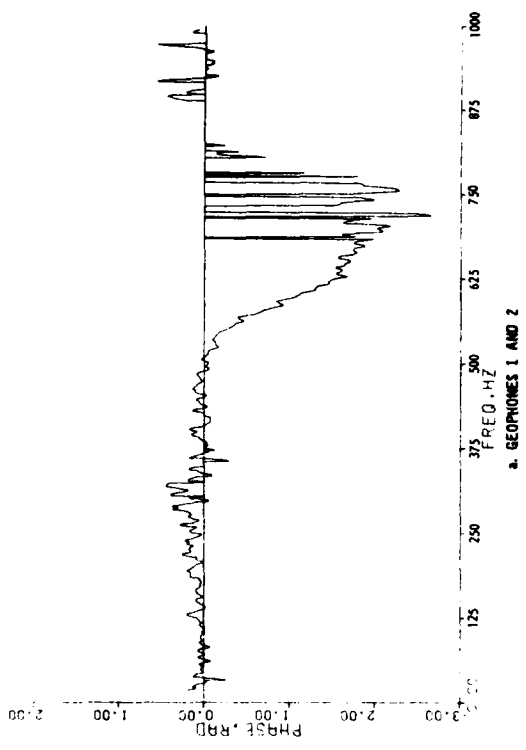Figure B2. Amplitude plots on shake table

B4

Figure B3. Phase difference of geophones
using impulse source method

all four geophones performed reliability up to 200 Hz. It can also be concluded that adequate data can be obtained between 200 and 550 Hz. Both tests concluded that geophone 1 had a greater chance of error above 550 Hz.

APPENDIX C: SOIL-CEMENT TEST AREA

1. The soil-cement test area is located within a large general test area at the U. S. Army Engineer Waterways Experiment Station (WES), Vicksburg, Miss.; the large area is described in paragraph 15 of the main text. The soil-cement area is a soil mixture of loess, sand, and cement placed in a pit 6.0 m wide, 6.5 m long, and 0.92 m deep. The original purpose of this test area was to simulate coral or frozen ground conditions by fabricating a soil mass that would produce shear and compression wave velocities of 600 and 1000 m/sec, respectively. With that purpose in mind, various materials were gathered, mixed, and placed in test cylinders until the desired mix or combination of materials was determined. The final product that was machine-mixed by volume contained 2/3 loess, 1/3 masonry sand, 6 percent cement (Portland Type 1), and 10 percent moisture. The soil-cement mixture was placed in the pit described above in 6-in. lifts and compacted using commercial power tampers. Each lift was compacted to an average density of 125 pcf.

Williams, Otis
  Rayleigh wave velocity measurements using broad band
frequency sources : final report / by Otis Williams
(Environmental Laboratory, U.S. Army Engineer Waterways
Experiment Station). -- Vicksburg, Miss. : The Station ;
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  42 p. in various pagings : ill. ; 27 cm. -- (Miscellaneous
paper / U.S. Army Engineer Waterways Experiment Station ;
EL-81-3)
  Cover title.
  "July 1981."
  "Prepared for Office, Chief of Engineers, U.S. Army
under DA Project No. 4A762730AT42, Task B/E3."
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Experiment Station. Environmental Laboratory. III. Title.
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